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ABSTRACT

This paper compares and contrasts cases of three beginning teachers, graduates of a teacher education program that included an intervention component designed to change prospective elementary teachers' knowledge and beliefs about mathematics education. The goal of the intervention (a sequence of three mathematics courses, a methods course, and a curriculum seminar) was to develop a more conceptual level of knowledge about mathematics itself and about the learning and teaching of mathematics. Each subject is described as both student and teacher of mathematics. Analysis focuses on ways in which knowledge and context influenced choices made by these novice teachers. Each new teacher faced responsibility for teaching multiple subject matter; deciding on the mathematical content children should have an opportunity to learn; creating mathematical tasks; and using instructional time given multiple and competing goals. Differences appeared in choices made in response to these issues. Choices were influenced by an interaction of views about knowledge and pedagogy and the degree to which context was perceived as a constraint. A question is raised regarding the nature of support required in the induction years if new teachers are expected to institute practices that are innovative and difficult to implement, questioned in traditional school settings, and unfamiliar to faculty, administrators, parents, and students. (Author/LL)

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INFLUENCING BEGINNING TEACHERS' PRACTICE IN MATHEMATICS EDUCATION: CONFRONTING CONSTRAINTS OF KNOWLEDGE, BELIEFS, AND CONTEXT

Sandra K. Wilcox, Perry Lanier, Pamela Schram, and Glenda Lappan

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Abstract

This paper contrasts cases of three beginning teachers. The teachers are graduates of a teacher education program that included an intervention component designed to change prospective elementary teachers' knowledge and beliefs about mathematics education. The intervention—a sequence of three mathematics courses, a methods course, and a curriculum seminar—aimed to develop a more conceptual level of knowledge about mathematics and the teaching and learning of mathematics.

The paper begins with a series of vignettes to sketch a portrait of each of the subjects as student and teacher of mathematics. In the discussion that follows, analysis focuses on the ways in which knowledge and context influenced the choices made by these novice teachers. An examination of the cases reveals considerable similarities as well as striking differences. The commonalities inhere in a set of issues that each new teacher faced: (a) being responsible for teaching multiple subject matters, (b) deciding on the mathematical content children should have an opportunity to learn, (c) creating worthwhile mathematical tasks, and (d) using instructional time given multiple and competing goals. The differences appear in the choices they make in response to these issues. The analysis suggests that the choices the teachers made were influenced by the interaction of their views about knowledge and pedagogy and the degree to which they perceived context to be a constraint.

The paper concludes with questions about the nature of the support that would be required in the induction years if new teachers are expected to institute practices that are innovative and difficult to implement, questioned in traditional school settings, and unfamiliar to faculty, administrators, parents, and students alike.

INFLUENCING BEGINNING TEACHERS' PRACTICE IN MATHEMATICS EDUCATION: CONFRONTING CONSTRAINTS OF KNOWLEDGE, BELIEFS, AND CONTEXT

Sandra K. Wilcox, Perry Lanier, Pamela Schram, and Glenda Lappan[1]

Recent calls to reform mathematics education (National Council of Teachers of Mathematics, 1989a; National Research Council, 1989) pose considerable challenges to those entrusted with preparing teachers—the primary agents of change in the nation's K-12 classrooms. Two recent documents of the National Council of Teachers of Mathematics ([NCTM] 1989a, 1989b) describe a vision of mathematics classrooms where students and teacher form a community of learners engaged with one another in inquiry; where teachers provide situations that lead to learner inventions; where students become mathematical risk-takers—making guesses and pursuing hunches, making conjectures and marshalling arguments in support of them; where the criterion for what makes sense is determined by students and teacher working together. Creating a vision of classrooms that enable mathematical inquiry and that empower learners and teachers is one thing. Preparing teachers with the knowledge and disposition to construct real classrooms that embody this vision is quite another.

The challenges that teacher educators confront are embedded in the deeply rooted ideas about teaching and learning mathematics that preservice teachers bring to their professional studies and the difficulty of professional study to overcome ingrained notions developed during previous school experiences (Ball, 1988; Feiman-Nemser, 1983; Tabachnick, Popkewitz, & Zeichner, 1979-80; Zeichner, Tabachnick, & Densmore, 1987). Preservice teachers' ideas about mathematics education have been shaped largely by their own experiences as learners of mathematics. Typically, they view mathematics as a linearly ordered, fixed body of knowledge that is best learned by memorizing facts, rules and formulas, and procedures for applying them to textbook exercises. They view the role of the

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[1]Sandra K. Wilcox, assistant professor, and Pamela Schram, instructor, in the Department of Teacher Education at Michigan State University, were senior researchers in the National Center for Research on Teacher Education (NCRTE), the forerunner of the National Center for Research on Teacher Learning. They are also researchers in the Elementary Mathematics Study, a longitudinal research project studying the change in preservice teachers' knowledge and beliefs about mathematics, what it means to know mathematics, and how mathematics is learned. Glenda Lappan, professor in the Department of Mathematics and former NCRTE senior researcher, is associate director of the project and principal designer and instructor for the sequence of mathematics courses in this study. Perry Lanier, professor in the Department of Teacher Education, directs the project and the Academic Learning Program from which the subjects of this study are drawn. The Academic Learning Program is an alternative teacher education program that emphasizes the development of a thorough understanding of school subject matters and a conceptual change view of learning and teaching. Each teacher candidate in the program has a unique field experience which involves working with a mentor teacher and a classroom of children each term over a two-year period.

teacher as carrying out goals determined by text material, providing demonstrations and examples of tasks to be completed, and checking assignments and tests for completeness and correctness. Preservice teachers bring with them pedagogical and epistemological orientations that conceive teaching and learning as matters of technical competence. They expect their professional studies to provide the techniques to make them efficient and effective teachers. If teacher educators are to cause prospective teachers to rethink these beliefs, we must create situations where these beliefs are faced and reconsidered.

The Elementary Mathematics Study

The Elementary Mathematics Study was conceived as an intervention in the Academic Learning teacher preparation program at Michigan State University. The goal was to demonstrate the feasibility of developing in prospective elementary teachers a more conceptual level of knowledge about mathematics and the teaching and learning of mathematics. In this intervention, teacher candidates enrolled in a sequence of three nontraditional mathematics courses devoted to exploring number theory, geometry, and probability and statistics.[2] A methods course and a curriculum seminar drew on the content courses and field experiences to engage prospective teachers in reconsidering their notions about mathematics education. The cohort of 23 students studied by the Elementary Mathematics Study entered their professional studies program in September 1987 and graduated in June 1989.

Data Collection and Analysis

Data for the entire cohort of teacher candidates consist of field notes of all mathematics class sessions and video recordings of some, as well as audio recordings of small-group work. Questionnaires were administered at seven points in the study. We collected samples of student work that included written assignments and exams. In addition, we followed an intensive sample of four students. Data from our intensive sample include tape-recorded interviews conducted at eight points during the program, observations of their student teaching, and interviews with their mentor teachers and fieldwork instructors. In the third year of the study, we conducted periodic observations and interviews of our intensive

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[2]Materials from the Middle Grades Mathematics Project (MGMP) were used extensively in the three mathematics courses. These materials, produced and field-tested under a grant from the National Science Foundation, have been published by Addison Wesley under the titles *Probability* (Phillips, Lappan, Winter, & Fitzgerald, 1986), *Similarity and Equivalent Fractions* (Lappan, Fitzgerald, Winter, & Phillips, 1986), *Spatial Visualization* (Winter, Lappan, Phillips, & Fitzgerald, 1986), *Factors and Multiples* (Fitzgerald, Winter, Lappan, & Phillips, 1986), and *Mouse and Elephant: Measuring Growth* (Shroyer & Fitzgerald, 1986). The materials use problem situations and multiple representations as ways to develop understanding of mathematical ideas.

sample in their first year of teaching to study both knowledge and contextual constraints in implementing a conceptual approach to elementary mathematics education.[3]

In a recent paper (Wilcox, Schram, Lappan, & Lanier, 1991), we argue that the intervention made a significant contribution to empowering prospective elementary teachers as *learners of mathematics*. Specifically, our data show an increasing reliance on the collective efforts of members within small groups at problem solving. We observed an increased willingness on the part of these preservice teachers to engage in mathematical investigations and an increased confidence in their ability to apply knowledge in unfamiliar problem contexts. They approached problems in various ways, offered multiple ways of investigating them, and argued the reasonableness of their conclusions. Over the two years there developed among the cohort a norm of collaboration, a valuing of different approaches to problem situations, and a shared responsibility for learning. Perhaps the most significant development among the students was the shift in the locus of epistemological authority—from a reliance on the teacher to their community of classmates and teacher, together using mathematical tools and standards to decide about the reasonableness of processes and results of investigations. Increasingly, students themselves judged the validity of the arguments they put forward.

Because the learning of mathematics was embedded in a context of learning to teach, developing subject matter knowledge could be linked to developing pedagogical content knowledge. Reflections on differences within the community of the teacher candidates themselves—how they learned, what they focused on, the questions they asked, the strategies they favored—helped them appreciate divergent views in the classroom and to talk about children's learning in more complex ways. They talked about group work, nonroutine problem situations, and multiple representations as powerful ways to explore mathematics and construct mathematical knowledge. However, in the context of their own teaching, as students teachers and then as first-year teachers, we uncovered a tension between an ideal vision related to themselves as adult learners of mathematics and their practice with young learners.

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[3]Earlier papers have provided a fuller treatment of the mathematical content of the courses as well as findings about the changes in these teacher candidates' beliefs and knowledge about mathematics education. See Lappan and Even, 1989; Schram, Wilcox, Lanier, and Lappan, 1988; Schram, Wilcox, Lappan, and Lanier, 1989; Wilcox, Schram, Lappan, and Lanier, 1991.

The Cases of Albert, Allison, and Denise[4]

Albert

Albert entered the Academic Learning teacher preparation program because he knew there would be an emphasis on mathematics, one of his weaknesses:

> I've typically set math as my objective. That's why I was in Academic Learning. I didn't like math, I didn't want to major in math, but Perry was there and I knew I would have to teach it, so I might as well work on it.

He also took some courses in the mathematics department even though they were not required for his program. Albert had some skepticism about the constructivist orientation he believed the program promoted:

> They think we should teach in a manner in which it makes it easier for a student to construct those ideas. They're saying a student who has to figure it out for themselves is more likely to remember. . . . In a way I disagree with that . . . because I don't think you always have to *discover* [emphasis his] it or do problem solving. I think you can be told some of it and still go through this construction process. I think you can make connections; sometimes those connections will even be given to you by a teacher. It's understanding the reasonableness that goes along with that learning.

How he first encountered an idea seemed to be less significant than having an opportunity to "figure it out for myself." In a final paper written for the curriculum seminar, he wrote that learners are always constructing knowledge, no matter what the teaching mode. For him the question was what kinds of experiences can help learners make more powerful connections in order to "construct bridges between that [new] information and other information in the mind that is related."

As a student in the mathematics classes, Albert often worked independently until he became stuck on a problem or until he had some insights about a problem that he thought might help others. What characterized virtually all his efforts was his determination to learn at a level *he* wanted. On several occasions he challenged what he considered an arbitrary procedure put forth either by another student or the teacher. The teacher typically invited Albert to make sense for himself, a challenge he always accepted.

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[4]Names of preservice teachers are pseudonyms.

Albert's field placement was in a third-grade classroom at an urban inner-city elementary school not far from the university. The district guidelines for mathematics emphasized computational speed and accuracy, daily drill to practice recall, and timed tests to assess learning of facts. Albert's mentor teacher had each student working individually at a particular computational skill. Some were working on addition facts, some on subtraction facts, some at subtracting with regrouping, some on multiplication facts. Three mornings a week, students took a one-minute test on facts. They were certified to move on to the next skill when they could complete 100 facts in five minutes with 100 percent accuracy.

While in the methods course, Albert created two units that were to be the core of his mathematics instruction during student teaching. He planned to use the minicomputer to help the students develop an understanding of subtraction and its relation to addition. However, his mentor felt that this would be an inefficient use of time. Instead, she wanted him to concentrate on "borrowing with subtraction" and multiplication facts. So Albert abandoned his plans to use the minicomputer even as he remained silently critical of her perspective. He did make a set of base-10 materials to demonstrate regrouping in subtraction, but most of the students did not make a connection between the concrete materials and the pencil-and-paper procedure they had been taught earlier.

Albert was allowed considerable latitude in teaching a unit on measurement, and in fact his mentor liked what he did. In an interview, she described her impressions:

> They started measuring things by using their feet and then paper clips and then pencils. . . . Then he proceeded to talk to them about the need for a standard unit of measurement. . . . Finally, they got rulers. . . . He brought in estimation first so the children could think about what it might be before they actually measured. It really was quite effective for them.

At the same time, the mentor indicated there were some trade-offs to this creative approach:

> This is a plus and minus, you know. . . . I am about two-and-a-half months behind. The geometry was wonderful, the measurement was wonderful, and there will be only 4 questions on the SAT on each of these and 150 on other things. . . . I would have wanted them to be into something more solid like addition, subtraction, and multiplication. . . . I'm extremely pleased at the concepts that were developed, what they can do that they never would have done and their enthusiasm. But they are going to be behind next year.

Albert was not particularly pleased with his efforts at teaching mathematics. He felt he needed resources of a different kind than his mentor was able to provide. For example, when he asked about problem-solving activities, his mentor referred him to the word problems in the text. He had rejected those problems, in part, because he wanted something that required "more thinking, with a lot of processes together." He felt the text presentation of ideas "was not enough for kids to really grasp what was going on," so he chose not to use it unless he "just didn't have any other ideas."

Before student teaching, he often talked about the value of students working together in groups, particularly the opportunity to share with others what each had learned. Yet in this context, he did not use small groups at all:

> My mentor does not use small groups. I just didn't want to take the time to teach them how to work in small groups. She is just more interested in getting through the material. She often tells me I am spending too much time on one thing. She sees Grade 3 as a time to expose, not to teach for understanding.

Albert is now teaching Grade 6 at an elementary school in a suburb of the nation's capital. He was recruited to teach in three different schools in the district and chose the one that most reflected the socioeconomic, racial, and cultural diversity of the community.[5] Albert's classroom is a wonderful place to visit. His enthusiasm for learning is infectious, his caring contagious.[6] At the beginning of the year, he arranged students into teams of four or five. Each team selected a name (one chose the name "Math Murderers") and each member of a team served as a captain at various times. These teams often engaged in an activity that required cooperation to complete the task. A first and lasting impression is that the students loved their teacher and enjoyed being in this classroom.

Albert was given a district curriculum guide for mathematics. The guide specifies 11 units and 99 objectives for Grade 6. In addition, the guide provides a large number of "instructional ideas" for each unit. At the end of the school year, Albert is required to

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[5]Several students in Albert's class are recent immigrants to the United States from Latin America, with limited proficiency as speakers of English. Other international students are from India and Pakistan. In addition, there are several African-American students in the class. Several students come from families receiving some form of public assistance. The other two schools serve communities that are predominately white and high income.

[6]Albert is fluent in Spanish and has used that knowledge to help the Latin-American students become comfortable in his classroom. In addition, he and these students are teaching the others some Spanish as a way to bridge the language and cultural differences among classmates. Albert believes this strategy is probably in conflict with a district policy of immersion in English for non-native speakers, but he has chosen to ignore the policy and would gladly defend his actions if questioned.

complete a form for each youngster indicating which objectives he or she has mastered and whether the student is below, at, or above grade level, or gifted and talented.

Initially, Albert focused his efforts on reading and social studies. He has always regarded these areas as his strengths, where he is naturally creative and sees ways to make connections among ideas. In addition, he felt he needed to convince the school's reading teacher and the other sixth-grade teacher that he could replace the district's basal with more interesting texts without hindering his students' progress in reading. As the year progressed, he gave increasing attention to mathematics instruction. He began to look for interesting problem situations and activities beyond what the textbook or the district guide provided. As he put it,

> The district guide gives you some activities, but there aren't any connections. It's a 20-minute activity. There's no build up to it and no follow up from it. Sometimes I don't have time to think through what I want to do with it afterwards.

Albert determined early on that among his students there was a wide range of achievement on computational proficiency with whole numbers and fractions. But he found that all the students seemed to do well with hands-on activities. He used various manipulatives with division and factors and multiples. Work on decimals was combined with running activities during physical education where students timed themselves with a stopwatch and then posed and solved problems related to the activity and the ordering of decimals. Measurement was included in several science units.

During our final set of observations, the students began a study of probability. Albert introduced the unit by asking the students to imagine they had been selected to test some games by a toy manufacturer. Their job was to play and analyze some games and to decide whether the games were fair. The first game he posed was tossing two coins. Player A scored one point if the two coins matched, and player B scored two points if the coins did not match.

He had pairs of students first practice tossing the coins to make sure everyone understood how the game was to be played and scored. He incorporated some mental arithmetic along the way with questions such as, "Suppose you had forgotten how many turns you had completed. Player A has 10 points and Player B has 10 points. How many turns would you have taken?" After each pair of players had completed one game of 20 tosses, he had two students collect data from the entire class. At this point, it was nearly lunch time. Albert told them to take out their journal:

> We're going to look at this game again tomorrow. In your journal I want to you to write three things: Is the game fair? If not, who has the advantage and why as compared to the other person? If it is not fair, how could you make it fair?

Evidence from what students wrote in their journals suggested that the only thing they considered in determining whether or not the game was fair was the awarding of unequal points to the two players. Albert asked the observer if she could suggest other games that might cause the students to reconsider and extend their reasoning. We considered two possibilities, a fair game where players did not receive the same number of points, and an unfair game where each player did receive the same number of points. The next morning they played a dice game in which Player A scored a point if the number rolled was prime, Player B scored a point if the number rolled was composite. This provided the opportunity for Albert to have them explore whether the point structure alone was a sufficient way to analyze a game. In our final conversation, Albert told us he planned to introduce students to tree diagrams[7] as a way to represent probabilistic situations. He also intended to incorporate some work with decimals, fractions, and percents in the work on probability.

Albert is particularly creative in incorporating mathematics with other content areas. He taught a series of lessons that merged data analysis objectives with a social studies activity, a "Treasure Hunt in Africa." The teams were supplied with a set of materials that included a world population data sheet, sets of graphs displaying mean monthly temperatures, and maps showing the status of independence, main economic activities, energy production and usage, and climatic and topographical features for various countries on the African continent. Accompanying the materials was a set of clues that could be solved by finding the necessary information among the various data sources and, when solved, ultimately led to the place where the treasure was hidden. The students engaged in the hunt with gusto, and there was considerable friendly competition among the teams. Albert had acquired the materials at an inservice workshop presented by the district, one of several he attended during the year.

Albert has a strong interest in the use of technology in teaching and at the urging of the principal has assumed responsibility to provide support to other teachers who want to

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[7]Sometimes called probability trees, tree diagrams are a way to represent the possible outcomes of a probabilistic situation.

make better use of the computer lab in the school.[8] His own students spend time several days a week in the computer lab working with various programs. Some programs focus on computational skills while others aim at developing problem-solving strategies. During one observation, students worked on a problem that develops skills at using a guess-and-check strategy. Some students were quite adept at using information from previous guesses to make a more informed subsequent guess, while others were not.

Albert chose not to step in and show the children how to refine their guesses. Instead, he rearranged the groups at several terminals so that they might learn from each other some strategies for improving their guesses. He admitted that this was probably not the most efficient use of time in the computer lab. But he defended his choice by arguing that he believed his students could and should learn from each other.

Despite the evidence that he is providing his students with interesting and challenging opportunities for learning, Albert remains critical of his efforts at teaching mathematics. He continues to be frustrated, as he was in student teaching, with what he feels is an unreasonable number of discrete objectives to be covered.

> The problem is, if I gave as much time to this [probability] as it really needs, I would have spent too much time on it. But I will, of course, spend the time it needs because I don't teach all the units. They told me to expose the kids. I said yes and then I went and I taught my stuff because I can't do it [just expose them to an idea]. Two weeks for each unit, that is what they told me. All those objectives, all the different activities were to be done in two weeks! There is no way I can do that.

Albert is disappointed with what he considers to be his lack of initiative in developing materials and searching out additional resources in mathematics. He believes the district has all the material resources he needs and it is up to him to make use of them. He did have a district mathematics specialist come to his class and teach a demonstration lesson, an experience he found valuable. In a final interview, he said he did not meet his own mathematics objectives for the year and is already thinking about the upcoming school year. In particular, he intends to use materials from the Middle Grades Mathematics Project because he is familiar with them. The materials embody many of the district's objectives and he thinks they are creative and will engage the diverse learners in his classroom.

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[8]Albert is developing a set of social studies curriculum materials that are computer-based. The development cost is funded by a grant awarded to the school for a proposal submitted by Albert and his principal.

Allison

Allison entered the number theory course using the language of the program—students can be "generators of knowledge," teachers need to "transform" their personal knowledge into something students can understand.[9] At the same time, when she spoke of mathematics, she said, "You start with the basic facts, go on to practice, and then real problem solving." She felt "rules are important" and that "computation practice is needed to get through certain rules." However, by the end of the number course, she was developing a different perspective about learning mathematics:

> I see the limits of my own learning by memorizing and now trying to recall the right procedure or knowledge. I need concreteness! I have trouble trying to conceptualize in my mind. I have to manipulate objects or draw it out. . . . [In the problems we have been doing] I'm always finding these different patterns and I feel like I discovered them. I feel good about myself, figuring something out for myself. . . . I like the way I'm not given *how* to do a problem. I figure out how to do it. When you are trying to work it out, you can verbalize with someone else the ideas you are thinking of. Are you thinking of it that way too [or] some way that I am not thinking about it? I think that really helps.

She began to have a sense that there are "big ideas that allow one to do lots of things." The fundamental theorem of arithmetic was one of those "big ideas" that she frequently went back to when working on a problem situation involving the structure of numbers. "It's not that important to be able to choose the right formula. If I know the reasoning behind how to figure something out, then I can find the rule."

Allison's field placement was in a middle school in a suburban district near the university. Her mentor teacher taught language arts and social studies to a class of sixth graders. As a result, Allison had no observational experiences in a mathematics classroom until student teaching. At that time, the program coordinator arranged for her to teach one hour of mathematics with another teacher in the building. In a conversation with us following her first week observing in the math classroom, Allison voiced a concern that the students might not accept the "way I want to teach math." The regular teacher's routine was to check the homework, go over the material in the day's lesson, and give time to begin the next homework assignment. Allison had planned her teaching around two of the MGMP

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[9]Students began their professional studies fall term 1987, and took the first of the three mathematics courses during spring term 1988. In the first two terms of the program, they were enrolled in a course on learning and a curriculum course.

units, *Mouse and Elephant* (Shroyer & Fitzgerald, 1986) and *Factors and Multiples* (Fitzgerald, Winter, Lappan, & Phillips, 1986). The first unit uses interesting problem situations and multiple representations to develop understanding of perimeter, area, surface area, and volume, and the relationships among them.

Allison grouped her students for activities but then did not capitalize on the materials or the grouping arrangements for their power to engage students in inquiry. For example, several days in a row she spent nearly the entire period writing formulas for surface area and volume at the overhead, plugging in numbers, doing the calculations, and having students copy this in their notebooks. When several students said they did not understand, she simply provided another example. In one instance, she had students use six cubes to build a rectangular solid of dimensions 3 x 2 x 1. The following exchange was recorded:

Allison:	How many days food supply is that package?
Ben:	6.
Allison:	OK, that's 6. How can we use the volume formula to figure that out? Look at your sheet.
Jane:	By counting the bottom front, bottom side, and height.
Allison:	So, how many?
Jane:	3, 2, 1.
Allison:	So what operation do I need to use?
Bob:	To see how many [sic] surface area?
Allison:	We're talking about volume. You need to pay attention, Bob. Julie?
Julie:	You add the base plus the front . . .
Allison:	You need to *multiply*, 3 times 2 times 1 equals 6, bottom front times bottom side times height and that will give you the volume. Now how can we figure out the surface area?

This continued for nearly 20 minutes with Allison questioning students, getting mostly wrong answers, and each time referring them to their sheet of formulas. The next day, in

an attempt to have the students see the connection of volume to something "real," she brought in a plastic container. She had the students think about how many brownies could fit in the pan. She told them the container measured 13 x 9 x 2. Students dutifully multiplied the dimensions and shouted out "234." "That's right," said Allison. Neither students nor teacher questioned what their answer represented and whether it made sense.

In a conversation with us, she was clearly disheartened with her efforts: "I get so frustrated. These classes are so short. I don't have time for the discovery mode. I feel like sometimes I just have to *tell* them, you know, tell them the formulas."

A key set of ideas in the unit is the relationship between surface area and volume and how these measures grow. Allison never did get to that. The teacher in whose classroom she taught encouraged her just to get the students to memorize and use the formulas to find surface area and volume when given a set of dimensions and not do "more of these extra kinds of things." In her journal, Allison wrote:

> I started with the discovery mode—BUT I'm seeing where I have difficulties teaching the "S" [students] how to discover. I feel I lack a direction to which to lead the "S." . . . I'm coming to the conclusion that I need to specifically teach them the formulas and allow them practice on working with figuring out the problems. And me guiding them on the how—actually pointing out where you "plug in" what dimensions. Maybe this goes against the grain of the discovery mode I'm trying to work with.

Despite her recognition that, for herself at least, memorizing rules and formulas had not been a very effective way to learn mathematics, she fell back on this practice as a student teacher.

Allison is currently teaching fourth grade in a small rural district. As part of her job interview, she had to teach a small group of fourth and fifth graders in the presence of several principals from the district. In preparation, she called us for some feedback on what she had in mind. She had some good ideas and some interesting activities, but she was not focused on the mathematics or what students might learn from doing them. We helped her focus on the central mathematical idea of the lesson—finding factor pairs for whole numbers—and then consider what activities and representations would help students develop an understanding of the idea.

Her final plan incorporated small-group work, using square tiles to create rectangles as one representation, cutting out rectangles and nesting them on a grid as another representation. She planned to have the students come back together as a group, sharing patterns they had discovered, making predictions about the continuation of the patterns, and

creating ways to test their predictions. Allison had some good ideas, but she needed help to push her thinking beyond just fun activities.

In her own classroom, she seems less concerned about providing opportunities for her fourth graders to engage in mathematical investigations. She does not have a district grade-level curriculum guide or a set of objectives, but she has been told by the principal and other teachers that they expect the students who leave her class to have mastered computational facts. To that end, she has students spend considerable time working individually on drill-and-practice and timed tests. She is particularly concerned about what she perceives to be a wide range of mathematical ability among her students.

In late winter, she implemented a self-paced, self-testing mathematics program that the fifth-grade teacher recommended as a way to deal with these perceived differences. Each student works individually on a set of isolated computation skills, has exercises checked by a student checker, and upon mastery of a skill (80 percent correct), moves on to the next set of exercises. Allison commented on her decision to use these materials:

> I started giving weekly timed tests because that is common among the teachers here. Sometimes I don't know why I'm doing it except for it pushes them to learn the facts because they want to do well on tests. The kids don't really complain too much [about the self-paced program]. They kind of like the feeling of doing math problems. They feel like they've accomplished something when they do a certain amount of problems and get them done.

Allison told us she uses the individualized skill development for three consecutive weeks and then has one week of problem solving. On one occasion we observed her students work in groups to find all possible pentominoes (see examples below).

The whole-class discussion that followed focused entirely on group processes to the exclusion of the mathematics they might have learned. In fact, when one of the students tried to talk about how he got his different shapes, Allison told him they would talk about that another

time. For now, they were to focus on how well they worked together and how they could improve.[10]

It appears from what we observed and what Allison told us that problem solving involves giving students some interesting activities that she thinks will be fun for them to do. She draws heavily on problem situations she remembers from the courses in her teacher preparation program. But these problem situations are used as isolated activities, the mathematics embedded in them is treated superficially, and the main purpose is to "give the kids a break from drill-and-practice."

During an observation late in the school year, Allison had the students work on an ecology unit. On this day they were given a data sheet on the per capita paper waste generated and recovered by a dozen industrialized countries. Students were given the task of computing with a calculator the percentage of waste recovered by each country. Allison demonstrated the key strokes that would yield the answer, reminding them of an earlier exercise in which they had worked with percents and money.

The students diligently carried out their calculations (although most needed some individual help in using the calculator properly), recorded their answers in the blank column on their data sheet, and answered questions about which country used the most paper, which had the highest recovery rate, and which recovered the most pounds. In a conversation following the lesson, Allison indicated she did not intend to do anything further with the lesson. We suggested she consider having the students make graphs as another representation of the data on waste generation and recovery.

When we returned for our final observation a week later, there were a number of colorful bar graphs on the bulletin board created from the earlier activity. What was particularly interesting was the variety of ways that the children had chosen to present their data. Some had displayed simple national comparisons of waste or recovery. Others had combined these features to make rather elaborate graphs. Teacher and students seemed particularly proud of their products.

Allison is not reluctant to ask for help from those around her. But at present, it seems doubtful that she has colleagues who can help her think about how to create a classroom where learners engage in mathematical inquiry. Considering the workshops her

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[10]At her principal's urging, Allison had attended an inservice on cooperative learning. The workshop presenter promoted a specific model: four members to each group with each member assuming a role—recorder, reporter, messenger, and timekeeper. She tried to use cooperative groups when she gave the children activities. In this instance, with the exception of the messenger, who got the activity sheets from Allison and distributed to group members, it was not made explicit what the others were to do, particularly the timekeeper. Allison moved around the room as the groups worked, clipboard in hand, making note of when children made socially appropriate and inappropriate comments to each other.

principal has had her attend and the kinds of suggestions she has received from colleagues, two issues are of foremost concern for her: how to manage the classroom efficiently and effectively and how to ensure the computational proficiency of diverse learners.

Denise

Denise came to teacher education with conceptions and beliefs about teaching and learning that she perceived would be advantageous as a teacher candidate. Like most people, especially college students, Denise believed she knew what teaching was. And like many who choose teaching as a profession, she had a parent, her mother, who was and is an elementary school teacher. Denise's conceptions of teaching were shaped in large measure by the time she had spent in her mother's classroom. She had been a regular visitor during high school and continued the practice following her graduation. In her interview prior to being accepted to the Academic Learning Program, she cited this experience as one of the things that might set her apart from other applicants. She continued this visitation while in college, because she thought it would help her in teacher education classes.

The opportunities to observe in her mother's classroom, along with her own experience as a student, had shaped her views about learning and how it takes place. In the first course in her professional studies, she wrote about an observation of a social studies lesson in her mentor teacher's classroom:

> The students were first led through a question-and-answer period on the region to get them thinking about what they knew. They then were given 15-20 minutes to complete the worksheet. Besides getting factual information about the region, this lesson also gave the students practice in skills they will use later in their schooling. . . . Most of the students easily disposed of the instructor's questions, but for those who needed help, the teacher led them logically through what they already knew, step by step, until they were able to deduct [sic] the answer, another valuable skill they can use in the future.

Although Denise was commenting on another teacher's instruction, much is revealed about her own sense of the essence of learning. Denise believed that one learned by accumulating facts and skills in a sequential, step-by-step process. The role of the teacher was to break a topic up into small pieces and provide sufficient practice so that students could climb the steps. In an interview midway through the program she told us:

> Math is hard to explore. I see math as moving step by step and a person needs to be able to climb the stairs. . . . I think, how can I break this down into learnable parts, break it down so that it is more teachable or easier to understand for the students. . . . Math boils down to all the functions—addition, subtraction, multiplication and division—as the foundation for problems.

The essence of learning from this instruction was remembering. During interviews, Denise manifested the degree of reliance she placed on memory. The following is a partial transcript of her attempt to find the least common multiple of 36 and 63:

> I know there is a divisibility test for 9, but I can't remember it, so. . . . Seems to me when we were in school there was a way to figure this out by breaking the two of them down into their factors. . . . 36 breaks down into 6 and 6 which breaks down into 3 and 2. And 63 breaks down into 9 and 7 and then 9 into 3 and 3. And then I can cross out numbers, but I don't remember how to do it.

In another problem she was asked to continue a sequence of figures and tell the perimeter of the *n*th figure. "It would be [pause] there's a formula if you go back to high school. I want to say length times width, but that's not it." She never seemed to be bothered when her memory failed her. She had been a successful student in mathematics, having completed a year of calculus, and was confident about herself as a mathematics student. She was equally confident of the adequacy of her mathematical knowledge to teach elementary school mathematics. Her experiences in our mathematics courses had little effect on challenging that confidence.

Denise did her student teaching in a fourth-grade classroom in an urban fringe district about 15 minutes from the university. The school was in the same district from which she had graduated and in which her mother taught. The district had a general curriculum director who assessed the mathematics curriculum in Grades 3-5 as pretty much textbook-driven. The district's fourth graders regularly scored above average on the state's assessment test of basic skills. The mentor's students were high within the district.

Denise's mentor agreed that Denise could teach the two units on number and geometry created in her methods course for student teaching. Her mentor was particularly interested in having Denise teach a unit on fractions. Denise created both units but never did teach the geometry unit. Instead, she followed the fraction unit (which emphasized equivalent fractions) with a series of lessons on multiplying two- and three-digit numbers by

one-digit numbers. That was followed by about a week of lessons on division of two- and three-digit numbers by one-digit divisors. When asked why she had abandoned the geometry unit for some lessons on division, she responded: "Division was the next logical progression in studying math. I didn't want to begin geometry because I thought it was better for the kids to move on to division rather than splitting up multiplication and division."

Denise's efforts as a student teacher were influenced by her belief that she knew how to teach and that her mother was the exemplar of the kind of teacher she wanted to be. This level of confidence meant that she seldom sought out her mentor or her university instructor for conversations about her work or assistance on the many occasions when things did not go well. She told the university instructor,

> I don't feel that there's anything you could have done, unless I would have come to you and said you know that I need help with this, or something like that. But what I really felt was that I needed to try and work a lot of it out on my own.

On the many occasions when the mentor or university instructor made suggestions about other ways to think about a lesson, Denise's response most often was to defend her actions as appropriate and other ways as less appropriate. Her mentor put it this way: "She felt that she knew a lot or if she didn't her mother did. . . . She wasn't willing to accept some suggestions at the outset, little ways." At the conclusion of student teaching, we interviewed all of our participants. One of the questions we posed was what weaknesses they felt they had as a teacher of mathematics. Denise responded, "I have none."

The step-by-step conception Denise had about the nature of mathematics was visible in her planning and instruction. Being a good student, she dutifully planned lessons that had students use manipulatives and group work as had been advocated in her mathematics and professional studies classes. However, the choices she made were driven by "neat" activities rather than an overall conception of the mathematical content or how the activity would help students understand an idea. Her choices were juxtaposed with her real goal for students of having them learn the steps in computational procedures. She consistently focused on "getting to the algorithm," as her series of lessons on division illustrates.

She began by having students put several hundred pieces of macaroni into 2, 3, and 4 groups. On the second day, she moved to the symbolic level, emphasizing place value and partial quotients, using this form as a way to "record the answer."

$$
\begin{array}{rrrrr}
 & 60 & & & \\
5\overline{)356} & 5\overline{)300} & & & \\
 & 10 & & & \\
 & 5\overline{)50} & & & 60 \\
 & & & \text{check:} & 10 \\
 & 1 & & & +\ 1 \\
 & 5\overline{)6} & & & 71\ \text{R}\ 1
\end{array}
$$

For class work, students were to get into pairs and divide 115 pieces of macaroni into 2, 3, 4, 5, 6, and 7 groups and record their answer as above. Most children worked with the macaroni. A few resisted, preferring to just do the problems symbolically.

However, these children very quickly ran into trouble. They could not figure out what to do in $3\overline{)115}$ when 3 would not divide exactly into 100. On the third day, Denise used a chip trading board to illustrate regrouping in division. Again, work at the concrete level was coupled with the symbolic record. On the fourth day, her final day of student teaching, she demonstrated the long division algorithm at the board.

In the span of four lessons, students had been given two different models for thinking about division as well as an algorithm. In the rush to the algorithm, she made little attempt to connect the various representations. In an interview following the third lesson, Denise explained her decision making:

> All I really wanted them to see out of that was an experience they could think back to when they get into the symbolic representation. I thought there were a few of them that would get the idea that if the 100 didn't divide evenly that they were going to have to do something with the extras. Now today, trading with the chips, I hope the connection is made. I hope that once it's followed through with the actual algorithmic step that this will all make more sense.

For Denise, getting to the algorithm was key because that would clear up any misunderstandings children had with the concrete models. Her mentor commented on this: "She's getting so much into the algorithms. She thinks this will make it clearer for the children. . . . To her, those symbols convey all the thoughts that she needs."

Upon completion of her professional studies, Denise was offered a teaching position in another state. She declined the offer, choosing instead to stay at home and work in her

father's business. At the beginning of the 1989-90 school year, she was offered a part-time position in a professional development school in the district where she had done her student teaching. Teachers from the school and teacher/researchers from Michigan State University were collaborating on classroom research that required reassigned time for teachers. Denise was invited to teach the mathematics classes of a third- and a fifth-grade teacher on a daily basis. She accepted this position with the understanding that she would be supported by and responsible to the university mathematics educator involved in the project and work in cooperation with the two teachers.

Denise brought to this setting the same confidence that she exhibited as a student teacher. She perceived herself as an especially innovative first-year teacher, in part, because she believed her teaching modeled what she thought her teacher preparation program had advocated—the use of manipulatives, small-group work, and problem solving. In contrast to Denise's self-perceptions, the teachers and the university mathematics educator developed a very different sense of Denise as a mathematics teacher. Although Denise did use concrete materials (e.g., the minicomputer with fifth graders, chip trading with third graders), her goal, as in student teaching, was to get to the algorithm, not to understand concrete models as another representation of an idea. In fact, Denise treated the manipulatives themselves in an algorithmic fashion. For example, the third graders learned how to manipulate chips in chip trading but when asked what they were doing and why, they had little understanding beyond moving objects around.

In an attempt to provide support to Denise, the university mathematics educator observed her teaching and talked with children to find out what sense they were making of mathematics. When these observations and informal conversations with children yielded data about the limitations of student understanding, the teacher educator provided feedback to Denise. That feedback consisted of questions to spark Denise's reflection on choices she made and tasks she gave students as well as suggestions about specific things she might try.

Just as in her student teaching, Denise resisted suggestions from others about how she might improve her teaching. In her mind, she was already doing the things that were being suggested to her. She would never acknowledge that there was a problem and any evidence to the contrary simply did not persuade her. She judged her teaching to be superior to veteran teachers in the building whom she regarded as traditional. She was proud of herself because she used manipulatives and thought that made her innovative and set her apart from others.

By the end of the first semester, the regular classroom teachers and the mathematics educator had become increasingly concerned about what she was doing to the children and

what they were not learning. Their worry that Denise was unable or unwilling to see the problems and their belief that things were out of control (management was a significant problem in the third-grade classroom) led them to raise the issue of whether or not she should continue teaching in their classes. At spring break, Denise was asked to resign.

Interaction of Knowledge and Context on Teacher Choices

A close examination of the cases of our three beginning teachers reveals considerable similarities as well as striking differences. The commonalities inhere in a set of issues that each new teacher faced: (a) being responsible for teaching multiple subject matters (Denise excepted), (b) deciding on the mathematical content children should have an opportunity to learn, (c) creating worthwhile mathematical tasks, and (d) using instructional time given multiple goals. The differences are apparent in the choices they made in response to these issues. Albert made choices that came closest to reflecting the pedagogical and epistemological orientation of his teacher preparation program. In contrast, Allison fell back on more familiar and traditional practices once she left the university for her own classroom. Denise, however, continued a practice established during student teaching and was unable or unwilling to acknowledge that her classroom was not a model of the vision her teacher preparation program had promoted. Our analysis suggests that the choices the teachers made were influenced by the interaction of their views about knowledge and pedagogy with the degree to which they perceived context to be a constraint.

Choices About Responsibility for Multiple Subject Matters

Albert and Allison were overwhelmed by the amount of preparation required to plan for and teach several content areas. Both said they spent 4 or more hours most nights preparing for the next day, and as much as 12 hours on the weekend. They tried to cope in different ways with a contextual constraint endemic to the work of most elementary school teachers—being responsible for teaching many school subjects.

In the beginning, Albert focused on the school subjects he considered his natural strengths—social studies and reading. He attended district-sponsored workshops that focused on these two school subjects. He sought out the assistance of curriculum specialists in the district. He was proud of the engaging, interesting social studies units he provided for his students.

Initially, his principle source of ideas for mathematics was the district curriculum guide. He was critical of the number of objectives he was expected to cover. He found the examples of specific activities of marginal utility. But he accepted the fact that he could not

"do everything at once." Only after he had become comfortable, though not fully satisfied, with his teaching of social studies and reading did he take a closer look at mathematics. By the second semester, he was using more problem-solving situations with his students and was trying to create sets of lessons that focused on some "big ideas." He had stopped worrying about all the objectives he was expected to cover. He believed that in-depth study of some key concepts was educationally more sound than superficial coverage of scores of discrete objectives and he acted on that belief.

Allison frantically tried to do it all and expressed frustration, even guilt, at not being able to create "fun activities" in all subjects. At one point she feared she was endangering her physical health by, in her words, "trying to be a Wonder Woman all in the first year." Allison did not have one or two content areas in which she felt particularly strong. Every subject was a challenge and she appeared not to have much in the way of district curriculum guides to help her make her way. Consequently, she did not build her own confidence or expertise in any content area.

None of the workshops she attended or videotapes she watched (at her principal's urging) were intended to develop her subject matter or pedagogical content knowledge. Rather, each instance focused on classroom management—how to group students for instruction, how to use cooperative group learning, how to use assertive discipline and proactive management techniques, how to implement generic teaching strategies for efficient teaching and learning.

Teaching only mathematics in the third- and fifth-grade classrooms did not create the same problem for Denise, but coping with two teachers and a university mathematics educator was a struggle for her. Her defense when things were not going well was, "It would be different if I had my own room."

Choices About What Children Should Have an Opportunity to Learn

Albert, Allison, and Denise were uncertain as to what should constitute the elementary mathematics curriculum, particularly in the early grades. Albert took a critical, questioning stance toward the district's grade-level objectives. Despite the claim in the curriculum guide that the objectives were "interrelated, rather than isolated," Albert found little in the guide to help him make the connections. He created sets of lessons around some big ideas—factors and multiples, probability, developing problem-solving strategies. He spent the time he felt his students needed to understand an idea rather than be held to some arbitrary schedule to get through the objectives.

Albert was civilly rebellious, as a student and as a teacher. But it was not a knee-jerk opposition to authority. Rather, he believed that just as learners could and should make sense of what they are being taught, teachers could and should make professional judgments about what they teach.

Before her student teaching, Allison said she "wished someone had helped [her] with the big ideas or a concept map" of the elementary curriculum. She was comfortable choosing the two MGMP units for her sixth graders during student teaching because "they were developed by experts and I've had experience using the materials." But with fourth graders she was less certain about what to teach.

Allison did not have an established set of beliefs about what children should have an opportunity to learn. She relied heavily on the advice of others. When other teachers and the principal told her they expected her students to enter the next grade having mastered paper-and-pencil computational algorithms, she took that as her cue to focus on arithmetic skills. Allison asked for and received suggestions about what to teach and how to teach. Without a set of beliefs to which she felt deeply committed, she did not have the capacity for or disposition toward assessing other's suggestions. Every offer of advice was as good as any other.

Denise's belief about the sequential nature of mathematics, particularly the mathematics of number, and her acceptance of the textbook as a bona fide account of this sequence caused her little worry about what to teach. Her decision to move to division and not teach her planned unit on geometry epitomized her confidence to make appropriate decisions on curricular matters.

Choices About Creating Mathematical Tasks

One of the most significant differences among our first-year teachers were the choices they made about mathematical tasks in which students were engaged and the materials with which they worked. Albert tried to create situations for students to think about key mathematical ideas, to see connections among those ideas, and to see the application of mathematical ideas in other contexts. He posed problem situations that did not lend themselves to obvious algorithmic solutions. He encouraged his students to talk about mathematics and he had them write in their journals about problems they were working on. He believed that students should be actively engaged in problem solving and that they could be interested in learning. Albert had a social and political awareness; he was committed to the diverse learners in his classroom and he valued their coming to understand mathematical problems.

Allison's earlier enthusiasm for the "discovery mode" had all but given way to a practice aimed at computational speed and accuracy, proficiency with conventional algorithms, and their application to routine word problems. When she did have students work on an interesting problem situation, her choice was driven more by a desire to give students a break from drill-and-practice than to have them engage in mathematical explorations. Rather than serving as a launch to study the mathematics contained within them or as an extension of key ideas, problems were simply presented as exercises to be solved.

Denise's learning goal, getting the steps right, stayed foremost in her thinking. Given what she knew about the algorithm and what the book emphasized, she would select or design tasks with models that were related. She did this for one of two reasons: they were supported by her university instructor or they made mathematics classes more interesting for students. But the bottom line, whether connected to tasks or not, was "the steps."

Choices About Using Instructional Time Given Multiple Goals

The choices the teachers made about how to use instructional time was influenced, in part, by the ways in which they were able and disposed to balance multiple educational goals. All three teachers wanted outcomes that evidenced subject matter learning and increased personal and social responsibility.

From our observations, it seemed Albert was particularly effective at weaving these two goals throughout instructional activities. Although students were grouped into teams of four or five, work in small groups often involved two or three on a team working together, while some worked individually. Albert often let the nature of the task posed and the desire of the students dictate the working relationships within the teams. But if he felt students could support each other more, particularly if a student had some knowledge that could help the others make sense of a problem, he would intervene. Most of the time teammates worked quite well together and whole-class discussions that followed rarely focused on group processes. Instead, talk centered on what students had found difficult or confusing about a problem, how they attempted to solve the problem, and what they had learned.

When Allison had her students work in small groups, there was an inordinate amount of time spent on developing certain behaviors for specific roles within the group—recorder, reporter, messenger, timekeeper. This particular model of cooperative learning was taken for granted. Allison seemed not to consider whether this arrangement was appropriate for all mathematical tasks. During each group activity we observed, students were to practice

a social skill (e.g., saying nice things to one another) while they worked on their math problem. As Allison moved among the groups, she noted examples of students engaging in the appropriate behavior. On not a single occasion did whole-class discussion that followed small-group work deal with the content of the problems children had been working on. Instead, the focus was exclusively on developing group norms.

Denise seemed to hold the view that subject matter learning was primary and that personal and social responsibilities were important only to the extent that they supported the learning of mathematics. She considered personal and social responsibilities more as means than as ends. In fact, her concern about using concrete models and small-group work was that it took so much time and might interfere with the material that needed to be covered.

The Influence of Context

The teachers in this study made different choices about what to teach and how to teach in their mathematics classes. Those choices were influenced by the interaction of several factors: (a) their view of knowledge—what it means to know, how one comes to know; (b) their knowledge of mathematics and beliefs about what should constitute the elementary mathematics curriculum; (c) their conceptions of effective mathematics teaching; and (d) the degree to which contextual factors—time, district curriculum guides, expectations of colleagues and supervisors, organizational features, the structure of teachers' work—were perceived as constraints.

Contextual factors did not significantly constrain the choices Albert made. On the contrary, it might be argued that he created his own context. He made the system work for him and his students, in part, by producing for the system. Albert negotiated his place among his colleagues. His decisions about how to use what others offered were shaped by a critical, questioning stance. The experience of others was an insufficient argument for Albert to try something in his classroom. What counted most was evidence and a fit with his beliefs.

Albert's stance toward knowledge and what it means to know—taking a critical perspective, relying on evidence and logic—were intellectual qualities and habits of mind that he brought with him to the preservice program. This stance was congruent with the epistemological orientation of the intervention. In Albert's case, it would appear that the intervention supported a view of learning and contributed to an emerging view of teaching that fit with a set of beliefs established well before his professional studies.

In the case of Allison, a new set of beliefs about teaching and learning mathematics began to emerge during her preservice professional studies. But they were insufficiently established or not held deeply enough to guide her in negotiating the complexities and competing demands of a first-year teacher. Allison did not hesitate to seek counsel or ask for help from experienced teachers, other novices like herself, or her principal. She had a strong work ethic and labored at implementing those practices that others said would help her manage her students as social beings and as learners with an assumed wide range of abilities. But she did not critically assess the advice given. What seemed to count was the degree to which there was consensus among those giving advice. Majority opinion and perceived usefulness were her yardsticks, not her own set of beliefs, logic, or evidence.

Like Albert, Denise's beliefs dominated her decisions and actions as a teacher. But unlike the others, Denise was not influenced in any way by the context in which she did her teaching. Context did play a significant role, but it was the primary context in which her prior knowledge about teaching and learning was constructed—her mother's classroom. The influence of this context on the acquisition of the prior knowledge, skills, and disposition that Denise brought to teacher education was so strong that her mathematical and professional studies could only produce tensions between the program's vision of mathematics classrooms and the classroom Denise was determined to construct.

Remaining Challenges

This paper has provided cases of three beginning teachers, graduates of a preservice intervention designed to develop in elementary teachers a conceptual understanding of mathematics and the knowledge and disposition to create classrooms where young learners actively engage in mathematical investigations. Our cases reveal the complexity of constructing classrooms where young learners create mathematical knowledge, where they engage in personal and group sense making.

As we reflect on these findings, we conclude that disciplinary study is necessary to develop in novice teachers a set of intellectual tools and a disposition to engage in mathematical inquiry themselves. But disciplinary study alone may be insufficient to overcome preservice teachers' deeply held beliefs about young children, what they can and should learn in the elementary mathematics classroom, and how they might learn that which is of most worth. Modeling new practices and nontraditional conceptions of mathematical pedagogy in the study of content may be insufficient to develop in beginning teachers the knowledge, skills, and beliefs to conceive of teaching as something other than telling or as more than a matter of technical competence.

Teacher educators need to consider what intellectual qualities and habits of mind teacher candidates bring with them to their preservice professional studies. One of our biggest challenges may lie in how to develop in preservice teachers a disposition to ask critical questions—about curriculum, instructional practices, educational policies, testing, their own learning and that of others, the contexts in which mathematics education takes place—the organizational features that structure daily life in schools for teachers and students. Teacher educators need to consider not only the subject matter and pedagogical knowledge constraints that may limit a new teacher's efforts at creating classrooms where students gain mathematical power, but also the contextual constraints that exist in real schools and how the new orientations to teaching and learning they construct are likely to be challenged.

We also need to consider what responsibility we have to provide support during the induction years for teachers who would institute practices that are likely to be questioned in traditional school settings. In the *Professional Standards for Teaching Mathematics*, the National Council of Teachers of Mathematics (1989b) argues for new models for the professional development of teachers:

> As teachers move into their first few years of teaching, much is at stake. Few current models used by universities, schools, and communities involve working together to support new teachers. Often the "umbilical cord" is cut abruptly, and the constraints of the real world of schools overwhelm the fragile perceptions these new teachers hold about what mathematics teaching and learning could be. The result is that many new teachers find it difficult to adapt what they have learned in their teacher preparation programs to the conditions in which they are teaching. (p. 5)

How can we extend the notion of community beyond the preservice program? What kinds of communities would need to be created among professionals in schools and how can we equip our students to be advocates of such communities? These questions deserve our serious and continued study and our best efforts at finding creative solutions.

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